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Mathematical Debate in the Classroom

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GEM

Workshop summary

1. Mathematical debates
2. Experiencing a mathematical debate
3. Report on some experiments in classes
4. Examples of some debate questions

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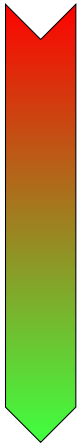

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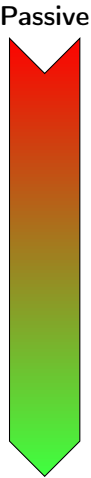

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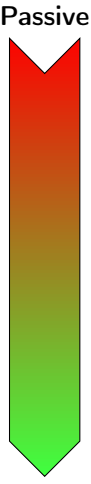

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- same as (3) but in a classroom situation.

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  Creating a habit/reflex to doubt oneself, to be cautious (e.g. an example is not the general case) translates into a related character trait : not judging people too easily, not jumping to conclusions

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- for the teacher :
  - Get a better idea of how each student thinks, of their strengths and struggles, etc.
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Endgoal : Structural shift in the way to approach the dialogue between teacher and students, e.g. by recognising the potential for fruitful **spontaneous debates** in reaction to some of the students' questions during the course.

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To get there : Practise this unusual setup of the debate in the classroom (both for the students and the teacher !) through **prepared debates**, so as to progressively develop a "culture of the mathematical debate".

In practice

Typical flow of a prepared debate :

1. Presentation of the debate subject/question
2. Individual reflection time
3. Private debate : discuss with your neighbour(s)
4. Proposing answers + vote
5. Public debate : defend your position and exchange arguments.
   We can switch back to private debate if/when necessary
6. Feedback from the teacher on the various arguments

Some "rules" for a fruitful debate

- Everyone can (but nobody is forced to) talk publicly ; everyone has to participate.
- When talking, one addresses the whole group, and not the teacher.
  - Look at the group as you talk
  - Announce your thesis ("I think that...") before giving your arguments ("because ...")
- Everyone is doing their best to listen and take into account the other's position.
  - Listen to the person taking the risk to express themselves (no private debate)
  - Look at the speaker
  - React to what other say with respect
- No argument of authority ("It's true because ... said so")
- Everyone can use the blackboard to explain their arguments

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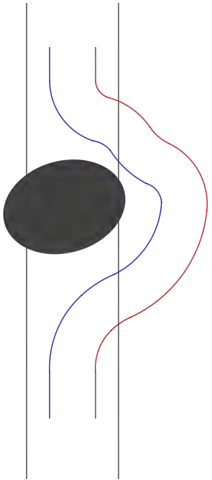

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Invidual reflection time
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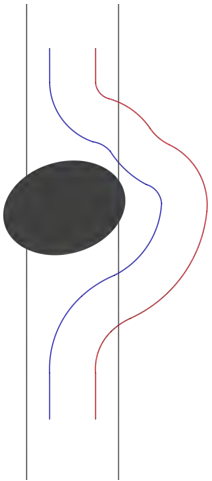

Let's clarify the problem

How do they move relative to each other ?

- Like a pair of oxen pulling a plow.
- Like bikers riding in formation side by side.
  Around the corner – https://www.youtube.com/watch?v=yYAw79386WI
  (around 1 : 55)
- Like two wheels on the same axle.

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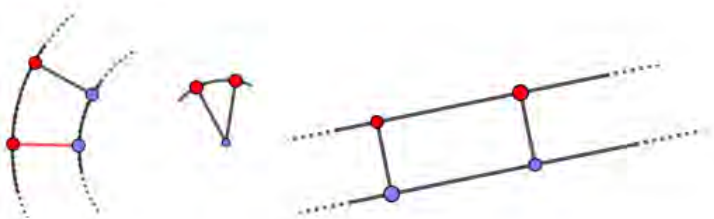

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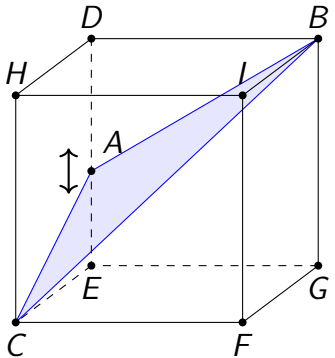
A spatial geometry problem

The figure below represents a cube. The point A can move freely on the edge $[DE]$.
What can one say about the triangle ABC depending on the position of A?
When will it be right-angled, isosceles, acute-angled, equilateral ...?



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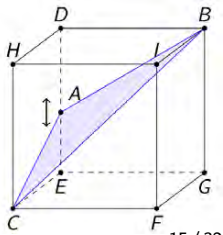
- Big motivation : the students come to the board, manipulate, exchange, ... They are very *active*.
- Usually quiet students get into the debate pretty well (and vice-versa !).
- Some difficulty to accept the scientific responsibility ("Will we get the *true* answers at the end ?").

Report on experiments : conjectures

- Students give enough conjectures (5 to 8).



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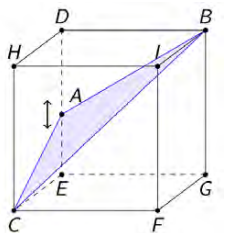

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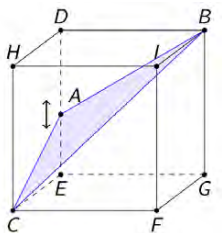

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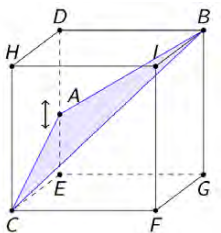
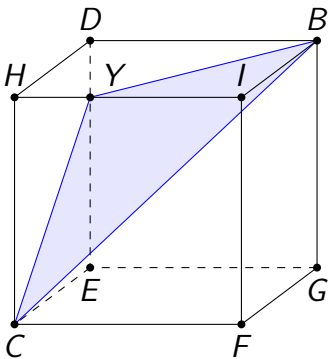
- Some unexpected conjectures.

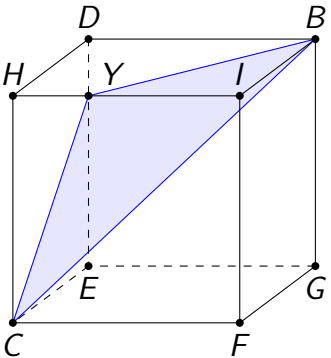


Example of an unexpected conjecture

A 5th year student (16 year old) thinks that when A is at "the intersection of $[DE]$ and $[HI]$", the triangle $ABC = YBC$ is isosceles.



Modify figures and isolate certain relevant elements



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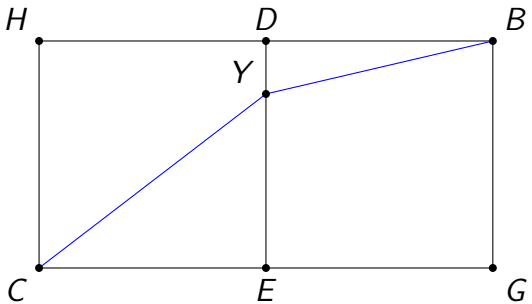

To compare the lengths of some edges, some students isolate faces of the cube.

Propose a better solution

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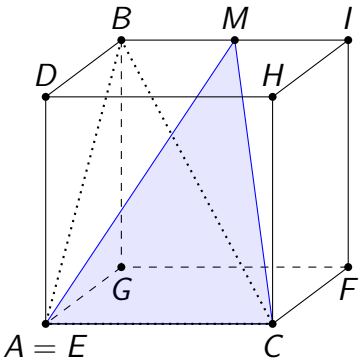
Another student counter-arguments : "But, don't you think it would be true if the point B was between B and I ?"



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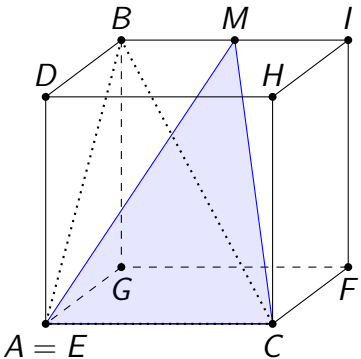
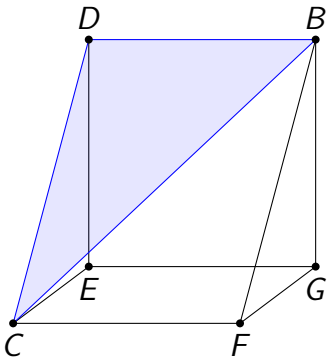


The student doesn't counter directly the argument, but circumvents it and proposes a better answer.

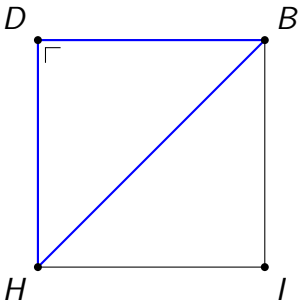
Complete a figure

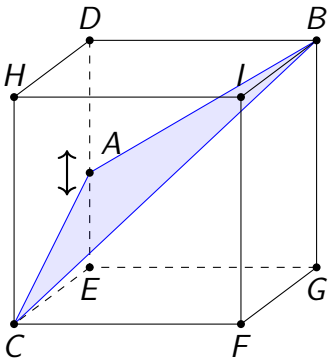
A student observes that the triangle BCD can be completed as a rectangle : $CDBF$. Hence, it is right-angled. He draws the figure on the right, slicing the original cube to make apparent the aforementioned rectangle.



Switch point of view

Some students propose to look at the cube "from above". They then have the (false) intuition that the triangle ABC will always be right-angled, because "the point A will always be in the depth of the point D".





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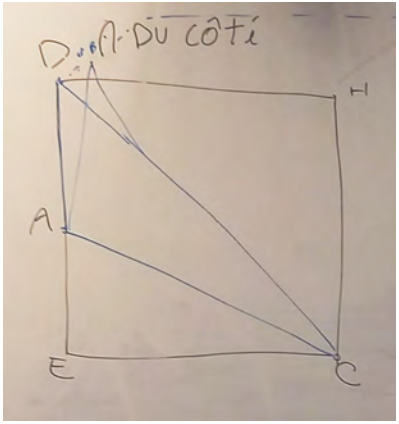
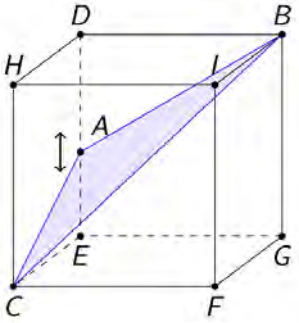


One could also consider the cube from another face. Here : the face *CEDH*, drawn by a student on the board.



Invoke false theorems

Argument of the perpendicular planes

"I think it is always right, because the two lines belong to planes which are perpendicular."
"My argument is not Pythagoras' theorem, but rather that the cube's faces are always perpendicular, hence the two edges, contained in perpendicular planes, must be perpendicular too !"

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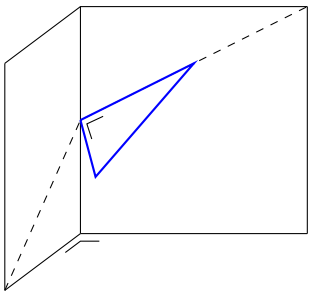
Implicit "Theorem"

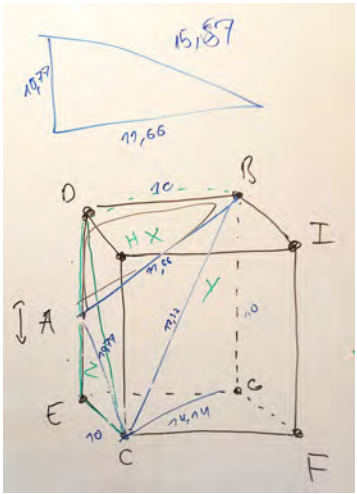
If two intersecting lines belong to perpendicular planes, they must be perpendicular to each other.

Manipulate objects in space



Empirical experience can also bring some elements for thought...

Invoking previous knowledge



A student, convinced that the triangle is always right-angled, computes the lenghts of the sides of the triangle ABC in a particular case, using Pythagoras' theorem... he shows the opposite of what he's aiming to prove.

Difficulties using previous knowledge

No transfer

In two 5th year classes with 6h/week of maths, students have just finished the chapter on vectoral geometry and scalar product.

Difficulties using previous knowledge

No transfer

In two 5th year classes with 6h/week of maths, students have just finished the chapter on vectoral geometry and scalar product.
They won't even mention vectors during the activity !

Students' words

Qu'as-tu pensé de cette activité de débat ?

- « *Elle était chouette et montrait que les mathématiques pouvait servir à autre choses que remplir des feuilles.* »
- « *C est très intéressant. Ça fait réfléchir chacun de son côté et puis permets de résoudre une question que une seule personne aurait sûrement pas réussi à résoudre seul.* »

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Qu'as-tu appris lors de cette activité de débat ?

- « *Qu'on pouvais persuader des gens de choses fausses si on avait d'asser bon arguments.* »
- « *Que il faut parler même si ton idée est complètement contradictoire à tout les autres car dans mon cas,j avais raison depuis le début.* »

Workshop summary

1. Mathematical debates
2. Experiencing a mathematical debate
3. Report on some experiments in classes
4. Examples of some debate questions

Examples of some debate questions

Spontaneous debate

Trigonometry – Understanding a new definition

When introducing trigonometric functions, one sees that the sin and cos functions vary between -1 and 1. What about the tangent ?

Conjectures debate

Analysis – Formulation of propositions

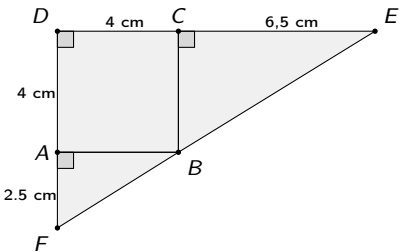
Write reasonable implications about functions, using the phrases "converges towards ...", "increasing", "which derivative is ...".

Example of some debate questions

Problem-debate

Geometry – Use diverse tools

Calculate the area of the figure below.



Joust

Statistics – Construct or use a concept

Which class did best on the test ?

A	B	C
8	2	7
9	3	8
9	6	8
⋮	⋮	⋮
13	13	15
13	16	16
15		18
		20

Example of some debate questions

True or false ?

Vectors – Interrogate and visualize a concept

"If the norms of two vectors grow, their sum also grows."

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